Proceedings of PSFVIP-4
June 3-5, 2003, Chamonix, France.
F4080

Structure and instability of a sink vortex

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KEYWORDS:
Main subject(s): *Dynamics of a sink vortex,*
Fluid: *Hydrodynamics,*
Visualization method(s): *PIV, LIF,*
Other keywords:*vortex*

ABSTRACT: PIV measurements and visualization by LIF (Laser Induced Fluorescence) were applied to a sink vortex of water (with a free surface) produced in a cylindrical tank rotating about the vertical axis. The controlling parameters are the rotating rate of the tank(0.2rad/s and 0.4rad/s) and the volume flux of a water withdrawn from a hole at the center of the bottom(50cm^3/s and 150cm^3/s). It was found that a Rankine-like vortex is produced in the steady state when the volume flux of a water withdrawn from the hole is 150cm^3/s. When the volume flux is 50cm^3/s, however, the conservation of the angular momentum around the central axis of the vortex does not established. The injection of fluorescence dye (rhodamine B) at the periphery of the cylindrical tank revealed that the water introduced at the periphery of the tank descends to the bottom along the side wall and flows to the center of the tank in the boundary layer at the bottom of the tank. However, the dye ascends in a thin vertical layer around the core of the vortex suggesting that the upward flow is formed around the sink vortex.

When we reduced a rotation rate of the tank from 0.4rad/s to 0.2rad/s while keeping the withdrawal rate of the water (to maintain a vortex), horizontal plumes appeared near the side wall because of the inertial instability. They were penetrated a limited extent in the interior region, so that we had a mixing layer near the periphery of the tank. At the same time ring-shaped disturbances developed in the interior region.

1. Introduction

The formation of a vortex near a small hole of drainage of a water tank is a well-known phenomenon (Lugt, 1979). It is explained by increase of the angular velocity of a converging water ring due to conservation of angular momentum. The first approximation of a sink vortex is the Rankine's combined vortex. It consists of an inner cylindrical core of radius R (say) in a solid rotation surrounding by a potential flow whose azimuthal velocity is proportional to r^{-1}.

In the actual sink vortex, however, there is a radial flow forced by suction of water from a bottom hole. In addition, the boundary layer is formed at the bottom of a tank. Khoo et al.(1997) observed the detailed structure of the boundary layer beneath a Rankine-like vortex. In their experiment, however,

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the suction was performed at the free surface. The role of the bottom boundary layer in their experiment is different from a sink vortex with suction at the bottom, because part of the inward flow in the bottom boundary layer is withdrawn directly into the hole when water is withdrawn at the bottom.

Another interest in the Rankine-like vortex is stability, because a free vortex with the velocity profile of r^{-1} is the marginal state for inertial instability. Having these motivations in mind, we observed structure and stability of a sink vortex produced in a laboratory tank.

2. Experimental device

2-1 Experimental tank

Fig. 1 shows the experimental tank used in this work. A cylindrical tank (made of plexiglass) with the diameter of 60cm with an inner vertical wall is placed on a ring-shaped bearing at the bottom of a big water reservoir so that it rotates about the central axis driven by a small motor. There is a hole with the diameter of 2.5cm at the center of the rotating tank. A water in the reservoir is sucked by a pump and introduced to the gap between the inner and outer side walls of the rotating tank. Water in the gap overflows and penetrates in the inner experimental tank with the diameter of 48cm through a sponge wall as shown in insert of Fig.1. Suction of the water through the hole is maintained by difference of water level between the experimental tank and the reservoir. The flow rate is measured by a flow-meter.

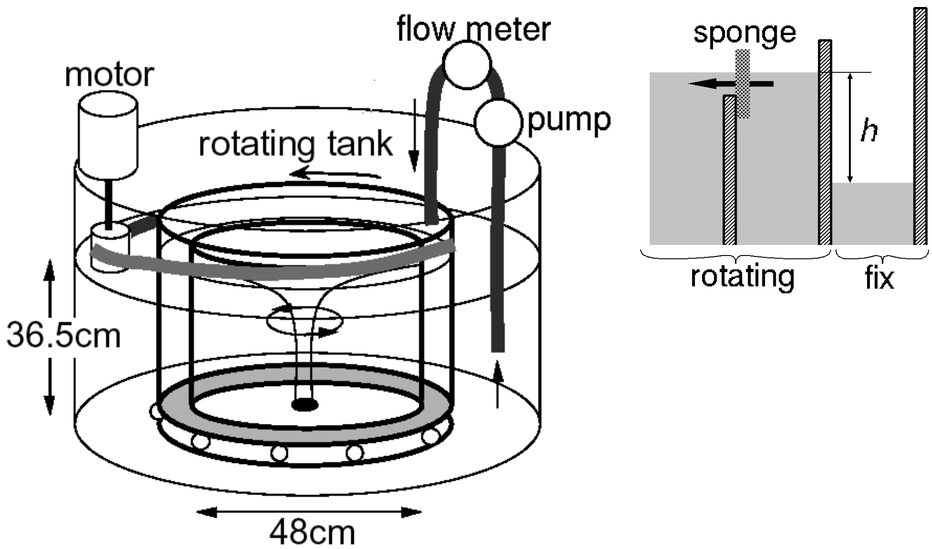

Fig. 1 Experimental tank. Insert in the upper right corner shows details of the periphery of the tank.

2-2 Observational set-up

Horizontal velocity fields at the height of 10cm from the bottom of the experimental tank were measured by a 2-dimensional PIV with a double-pulse Nd:YAG laser. Since the free surface of the water is curved, particle (talcum powder) images were observed through the bottom of the tank. Since our PIV cannot observe the vertical velocity fields (except non-rotational case) , we performed visualization of the flow fields by introducing fluorescence dye (rhodamine B) at the periphery of the rotating tank (by injecting dye at the top of the sponge wall). We observed spreading of the dye by a digital video through a color filter so that only dye is visualized.

2-3 Experimental procedure

The controlling parameters are the rotating rate of the tank and the volume flux of a water withdrawn from a hole at the center of the bottom. We selected 2 cases for each parameters. The combination of the parameters are shown in Fig. 2. The PIV measurement was performed in the steady state. Instability of the vortex was observed by reducing the rotating rate from 0.4rad/s to 0.2rad/s.

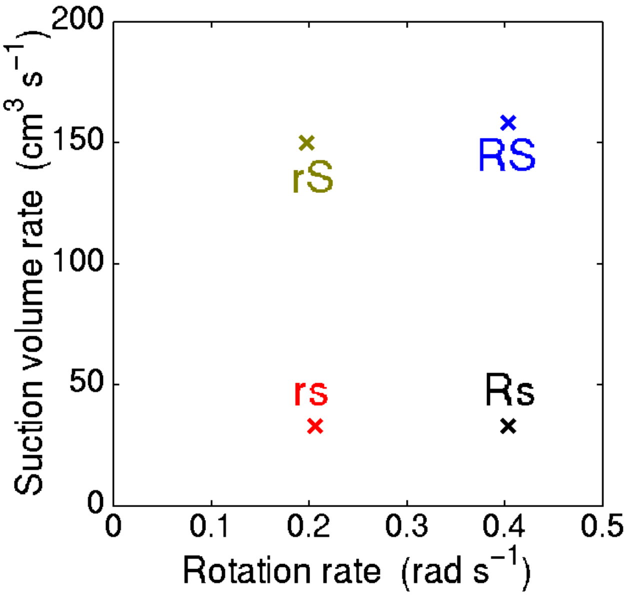

Fig. 2 Parameter space and names of 4 cases observed in this experiment

3. Results and discussion

3-1 Azimuthal velocity profile

Fig. 3 shows an instantaneous flow field observed by PIV. It shows almost steady Rankine-like vortex. However, detailed observation revealed that the position of the vortex axis fluctuates near the center of the rotating tank. Fig. 4 shows a trajectory of the center of the vortex at every 0.1 rotation of the tank based on the results of 10 successive measurements. To reduce errors due to this fluctuation, results of the PIV observation is transformed into a cylindrical coordinate referred to the center of the vortex and then we obtained the averaged flow fields during 1 rotation.

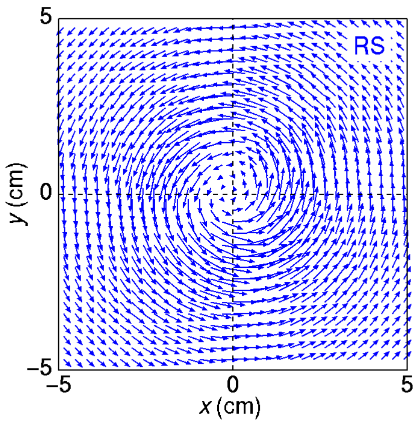

Fig. 3 Instantaneous horizontal flow field at the height of 10cm from the bottom in the RS case

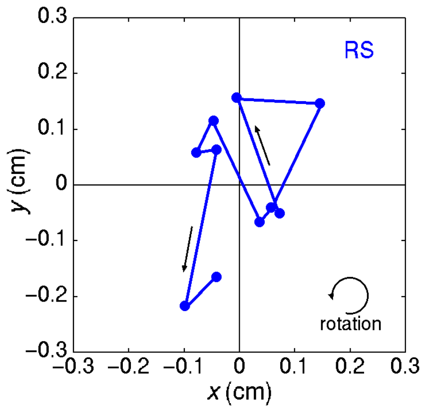

Fig. 4 Trajectory of the center of the vortex at the height of 10cm in the RS case

Fig. 5 shows the averaged azimuthal velocity profile obtained in this procedure. Broken lines in this figure show the velocity profiles expected if the angular momentum of a fluid ring given at the periphery of the tank is conserved (i.e., ideal Rankine's vortex). Note that observed velocity profiles are nearly the Rankine's vortex only when the suction rate is large. This results also show that the radius of the core (R) is a little greater than the radius of the hole. R depends on both rotation rate and suction intensity. The conservation of the angular momentum is checked in Fig. 6 where the radial distributions of the angular momentum are shown for 4 cases. Note clear deviation from the Rankine's vortex.

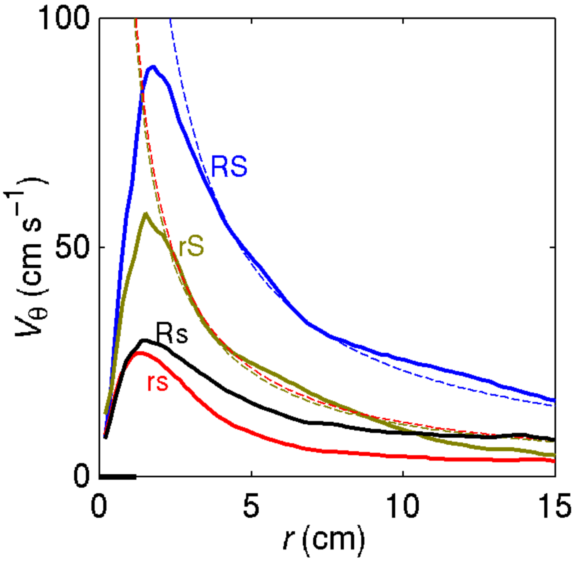

Fig. 5 Average radial profile of the azimuthal velocities of 4 cases. Broken lines in the figure show the azimuthal velocities expected from the conservation of angular momentum. The thick line at the bottom shows the radius of the hole.

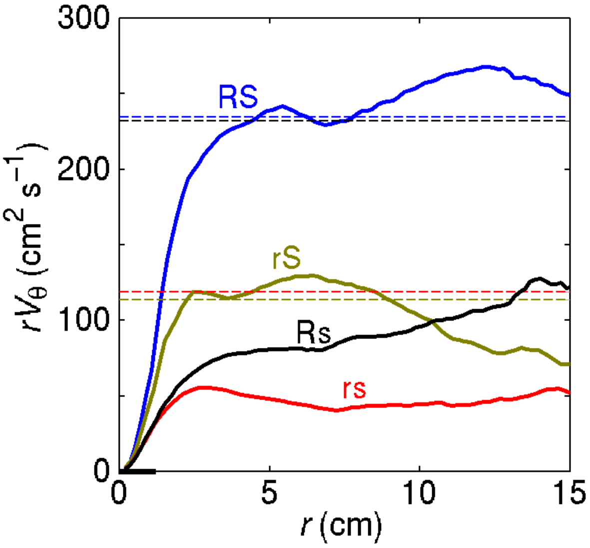

Fig. 6 Average radial profile of the angular velocities of 4 cases. The thick line at the bottom shows the radius of the hole.

3-2 Radial velocity profile

Fig. 7 shows the average profile and the standard deviation of the radial flows observed in RS case. The broken line shows radial flow expected when the inward volume flux is constant. Since the average flow is very small and the standard deviation is very large, we cannot get any definite conclusion from this result.

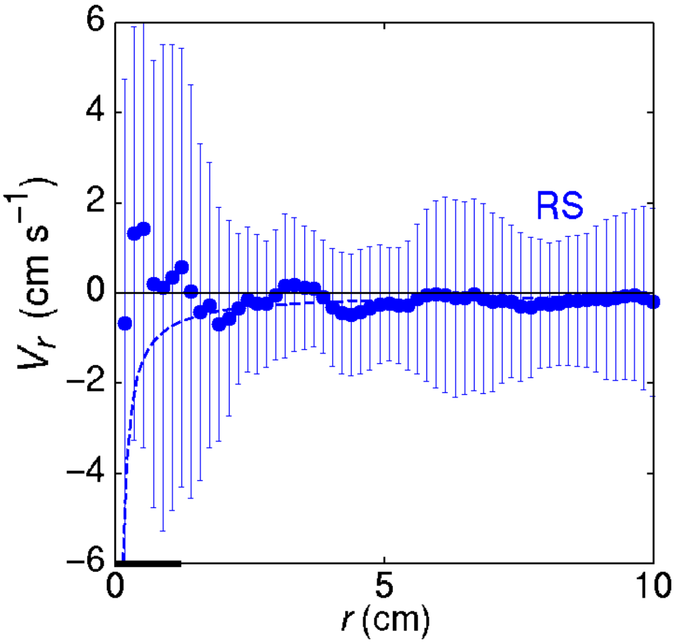

Fig. 7 Average radial profile of the inward flow at the height of 10cm. Broken line shows the radial flow profile expected from the conservation of the volume flux.

3-3 Vertical velocity field

Fig. 8 shows distribution of dye in almost steady state for the RS case. Note that the penetration of dye in the interior region is limited. The introduced dye descends along the side wall and directs to the center of the vortex in the bottom boundary layer. Most striking result in this experiment is that there is an ascending flow near the core boundary, suggesting that part of the inward flow in the boundary layer recirculates in the interior region and other part is withdrawn into the hole. We also observed the downward volume flux along the axis of the vortex is small. This results suggests that most of water introduced in the tank goes out through the hole not via the body of the vortex, but via the bottom boundary layer.

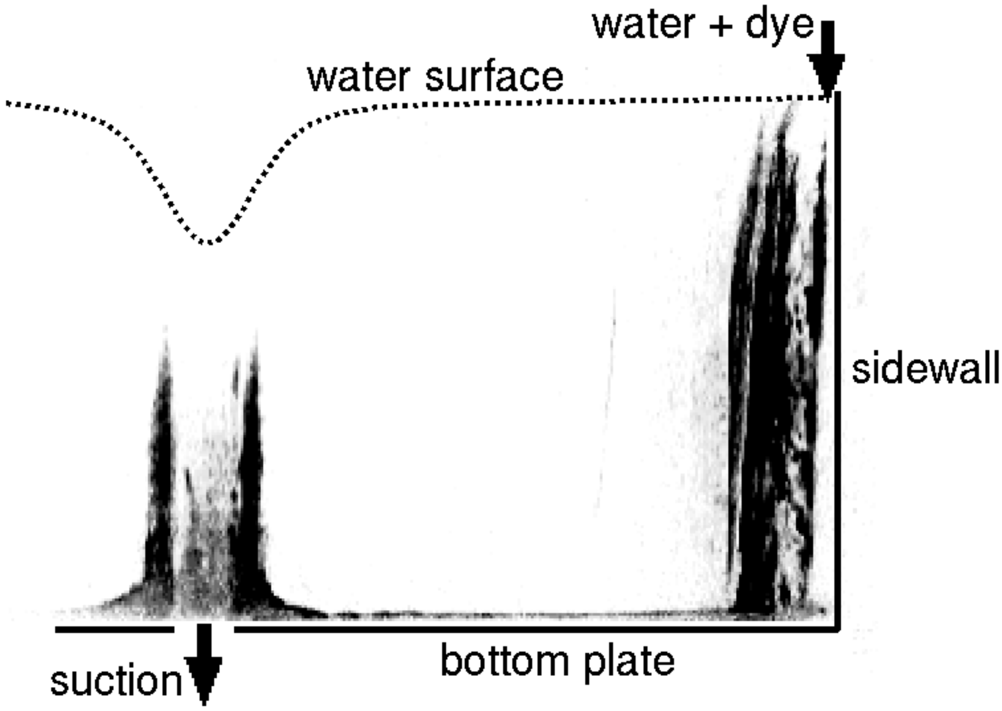

Fig. 8 Visualization of fluorescence dye introduced at the periphery of the tank. This pattern was obtained in the steady state by the continuous injection of dye.

3-4 Instability of a sink vortex

Instability takes place when the azimuthal velocity of the central region of a vortex is much higher than that of the outer region. This instability is well demonstrated in a fluid layer contained in a gap of concentric two cylinders when the inner side wall rotates faster than the outer side wall. The secondary flow produced by this instability is Taylor vortices. It is known that the marginal state for this instability is given by the r^{-1} distribution of the azimuthal velocity which is realized in the Rankine's vortex.

To realize this instability we reduced the rotation rate of the tank from 0.4rad/s to 0.2rad/s in the RS-case. Then, plumes appeared and grew from the side wall as shown in Fig. 9. Penetration of these plumes into the interior region, however, was limited so that we had an almost steady mixing layer along the side wall. At the same time dye introduced in the bottom boundary layer started to penetrate into the interior region with regular intervals in the radial direction as shown in Fig. 10. The penetration of the dye suggests that the secondary flow in the form of convective cells are produced in the interior region. The mechanism of this secondary flow is not clear at the present stage. Probable cause is the inertial wave excited by the mixing at the side wall or some kind of instability of the bottom boundary layer.

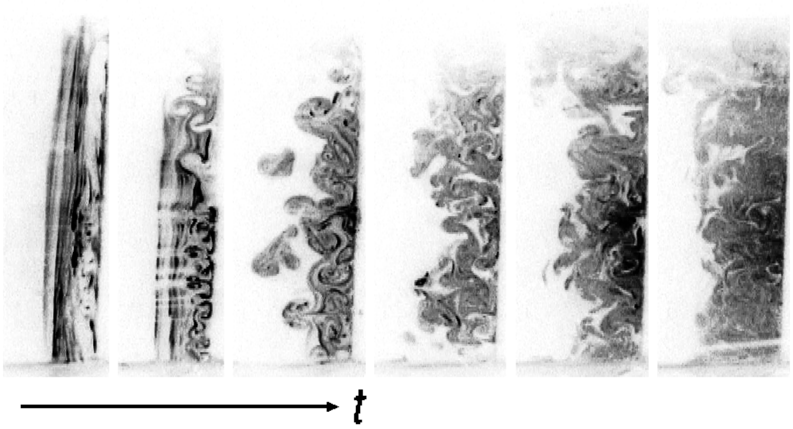

Fig. 9 Development of plumes at the side wall produced when the rotation of the tank is reduced. The time interval is 5s.

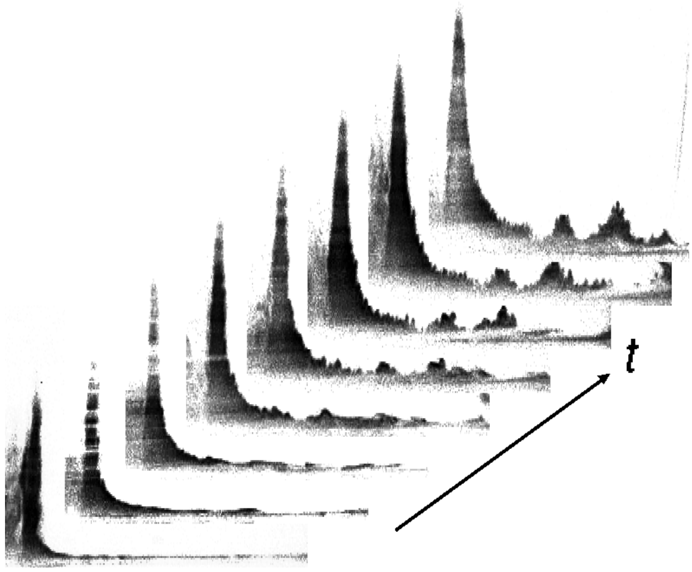

Fig. 10 Penetration of dye into the interior region after the rotation of the tank is reduced. The time interval is 5s.

Conclusion

PIV measurements and visualization by LIF were applied to a sink vortex produced in a laboratory to find following results.

1) Similarity to the Rankine's combined vortex depends on suction intensity. When the suction intensity is weak, conservation of angular momentum in the radial direction is not good.
2) An upward flow accompanies the sink vortex near the core region. Most water introduced in the periphery of the tank gets out through the hole through boundary layers formed at the side wall and the bottom.
3) When the rotation of the tank is reduced, plume-like disturbances are produced near the side wall. At the same time the secondary flow in the form of convective cells are excited in the interior region.

References

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